

$$I_1 + I_2 = I_3$$

$$\frac{V_{R1}}{R_1} + \frac{V_{R3}}{R_3} = \frac{V_{R2}}{R_2}$$

Since we know V_{S1} , we can state that

$$V_{R1} + V_{R2} = 15V \text{ or } V_{R1} = 15 - V_{R2}$$

For loop V_{S2} , we can state

$$V_{R3} + V_{R2} = 12V \text{ or } V_{R3} = 12 - V_{R2}$$

Then using our V/R current statement

$$\frac{15 - V_{R2}}{10} + \frac{12 - V_{R2}}{1} = \frac{V_{R2}}{5}$$

Multiply each term by 10 to remove the denominators

$$\left(\frac{15 - V_{R2}}{10}\right) \frac{10}{1} + \frac{10}{1} \left(\frac{12 - V_{R2}}{1}\right) = \frac{10}{1} \times \frac{V_{R2}}{5}$$

$$15 - V_{R2} + 10(12 - V_{R2}) = 2V_{R2}$$

$$15 - V_{R2} + 120 \overset{\text{Simplify}}{-10V_{R2}} = 2V_{R2}$$

$$135 = 13V_{R2}$$

$$V_{R2} = \frac{135}{13} = 10.38V$$

I_2 is current through R_3 , and I_3 is current through R_2 and reference node

Also note that voltage V_{R2} is of special interest, since it is associated with the reference node, V_{R2} is the voltage drop of R_2 caused by I_3

Therefore

$$I_{R_2} = \frac{10.38}{5} = 2.076 \text{ A}$$

we can then infer the rest

$$V_{R_3} = 12 - 10.38 = 1.62 \text{ V}$$

$$I_{R_3} = \frac{1.62}{1} = 1.62 \text{ A}$$

$$V_{R_1} = 15 - 10.38 = 4.62 \text{ V}$$

$$I_{R_1} = \frac{4.62}{10} = 0.462 \text{ A}$$

Plugging back in to check

$$V_{R_1} + V_{R_2} = V_{s1}$$

$$4.62 + 10.38 = 15 \text{ V} \quad \checkmark$$

$$V_{R_2} + V_{R_3} = V_{s2}$$

$$10.38 + 1.62 = 12 \text{ V} \quad \checkmark$$